This discussion of sources of error due to real changes in refraction—the effect of which can be determined and accounted for—is of more importance from a meteorological and, probably, even from an astronomical point of view than the many discussions in present-day astronomical literature regarding the failure of methods of observation due to accidental refraction errors, to sluggish levels, and to various instrumental errors. It is, therefore, wholly appropriate that it comes from an observatory whose first director[14] was moved to write:

> The observatory is not primarily designed for a meteorological station. Its very exceptional situation, however, creates a responsibility on its part to engage to some extent in making * * * meteorological observation * * *. The elevated and isolated site of the observatory will render researches on astronomical refraction of especial value, and the disposition of buildings and instruments has been made with this end in view.—*Wm. G. Reed.*

ON THE ABNORMAL PROPAGATION OF SOUND WAVES IN THE ATMOSPHERE.[1]

By S. Fujiwhara.

[Abstracted for the Review, by *H. Bateman*, Ph. D., Govans, Md.]

1. Shape of the region of audibility.

Peculiarities in the shape of the region of audibility of the sound from an explosion have been noticed very frequently during the last few years and have been associated with various meteorological conditions. The present war has provided splendid opportunities for the study of the propagation of sound waves produced by cannonading, and several papers have been written on the subject. In Japan the frequent explosions and eruptions of Mount Asama have provided even better opportunities for investigation and Japanese scientists have collected some very valuable information and made systematic observations. The material thus obtained has been studied mathematically by S. Fujiwhara in two able memoirs in which he shows that important information with regard to the structure of the atmosphere can be derived from a knowledge of the shape of the region of audibility. Some interesting types of regions are shown in figures 1 to 6.

A brief account of Fujiwhara's first memoir has already been given in this Review (May, 1914, **42:** 258–265). In his second memoir the author uses his mathematical results to obtain an idea of the shape of the region of audibility for each of the five types of atmospheric structure described by C. J. P. Cave in his book The Structure of the Atmosphere in Clear Weather. These are:

(*a*) "Solid" current, in which the wind remains steady in both direction and velocity in the upper layers.

(*b*) Continued increase of velocity beyond that of the gradient wind.

(*c*) Decrease of velocity in the upper atmosphere.

(*d*) Reversal or a great change of direction in the upper layers.

(*e*) An upper wind blowing out from a distant low-pressure center; frequent reversal in the lower layers.

For all the cases in each class Fujiwhara has examined whether a discontinuity of the region of audibility of an explosion can occur or not; his results are shown in Table 1.

Table 1.—*Classification of the regions of audibility about an explosion center for the different cases of atmospheric structure described by Mr. Cave.*

Type.	Number of cases.					Total.
	a.	*b.*	*c.*	*d.*	*e.*	
N_t	30	48	27	34	36	175
N_d	0	21	0	20	23	64
N_d/N_t	0	0.44	0	0.59	0.64	0.37

Note.—N_t denotes the total number of cases in each class; N_d, the number of cases in which discontinuity is to be expected.

Thus, if the region of audibility is discontinuous the winds in the upper atmosphere can not belong to classes *a* or *c*, *so long as horizontal homogeneity of the atmosphere is assumed.* In nearly all cases greater ranges of rays correspond to smaller gradients of wind velocities—air temperatures and the inclinations of the rays at the start being assumed as given. Since the direction of the axis of the region of audibility coincides nearly with that of the relative wind velocity at the height where the reflection takes place (see §3, eq. (3)), we can get a rough idea of the wind direction in the upper atmosphere from a knowledge of the surface wind and the direction and range of the regions of audibility.

In the first memoir the author pointed out that the occurrence of the detached region of audibility, i. e., an abnormal region, was closely connected with the existence of powerful cyclones. Many cases given in the present paper, however, are not in keeping with the above conclusion. The cases given in the first paper, however, occurred chiefly under weather conditions of the winter type. Now in winter the direction of the monsoon in Japan is northwesterly and no local depression of importance occurs over central Japan. Thus the region of audibility must continuously extend toward the east or southeast under the normal condition of winter. But when a powerful cyclone approaches, the above condition is disturbed, and detached regions of audibility may be detected. In the summer months the direction of the monsoon is southeasterly, and the development and passage of a local depression over central Japan is a daily phenomenon. The direction of the upper wind due to this depression, whose height above sealevel is comparatively small, about the same as that of Mt. Asama, can by no means coincide with that of the monsoon, and hence detached regions of audibility may occur. When a powerful cyclone approaches from the west, we may have abnormal cases in which no detached region can be detected or in which one is found in a direction other than westerly. In the period of transition from the weather of winter to that of summer, or vice versa, the direction of the wind of monsoon type is not confined to the northwest or the southeast, but can be anywhere between the northwest and southeast on the north side. In these periods the energy of the monsoon becomes less and the effect of cyclones or other meteorological elements on the phenomenon of the propagation of sound waves can easily become predominant.

The mathematical theory is worked out on the assumption that the atmosphere is uniform in each horizontal plane. There must also be further abnormalities of propagation when the uniformity is broken by the presence of floating sheets or masses of cloud in which the temperature may be different from that of the surrounding air and when a Helmholtzian wave exists at the boundary of two layers of the atmosphere with different velocities

[14] [*Holden, E. S.*]: Description of the meteorological instruments. Univ. Calif. publ. Lick observ., 1:78 Sacramento, Supt. State Printing. 1887.

[1] *Fujiwhara, S.* On the abnormal propagation of sound waves in the atmosphere. Second part. Bull., Central met'l. obs'y. Tokyo, Japan, 1916, 2, pt. 4. pl., 32 p. 27 lith. 4°.

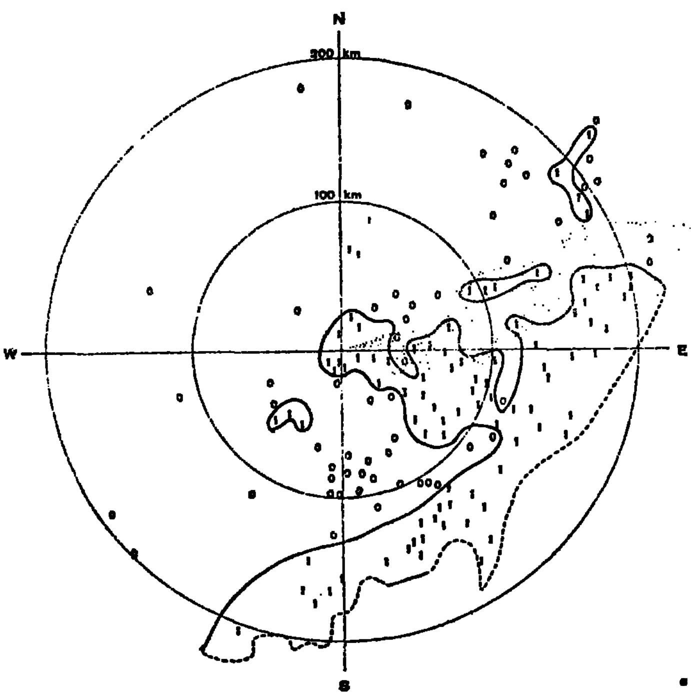

FIG. 1.—Regions of audibility of sounds of explosion of Asama volcano, Dec. 14, 1912. (Number of reports heard indicated by figure at each station.) Ashes fell over the stippled area. (Author's Chart Ia.)

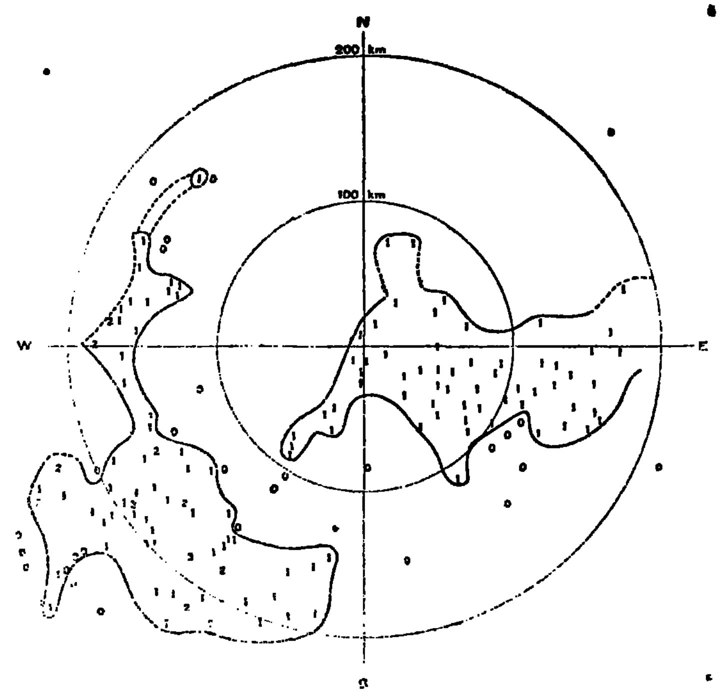

FIG. 2.—Regions of audibility of sounds of explosions of Asama, June 17, 1913. (Author's Chart VIa.)

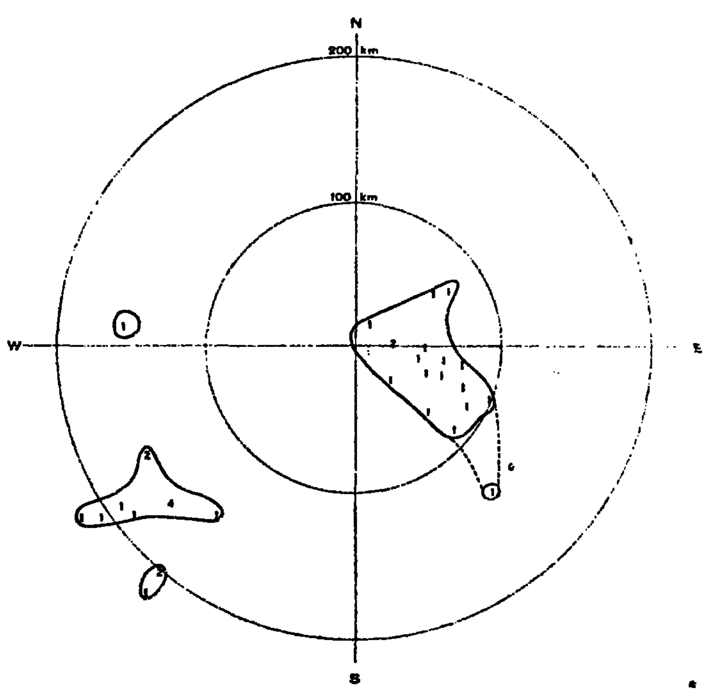

FIG. 3.—Regions of audibility of sounds of explosions of Asama, Aug. 12, 1913. (Author's Chart XIVa.)

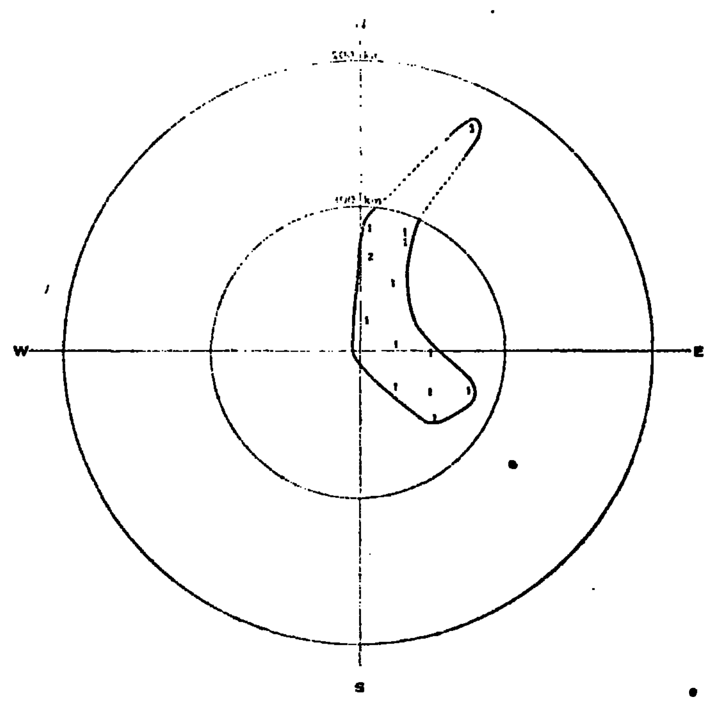

FIG. 4.—Regions of audibility of sounds of explosions of Asama, Oct. 26, 1913. (Author's Chart XVIIIa.)

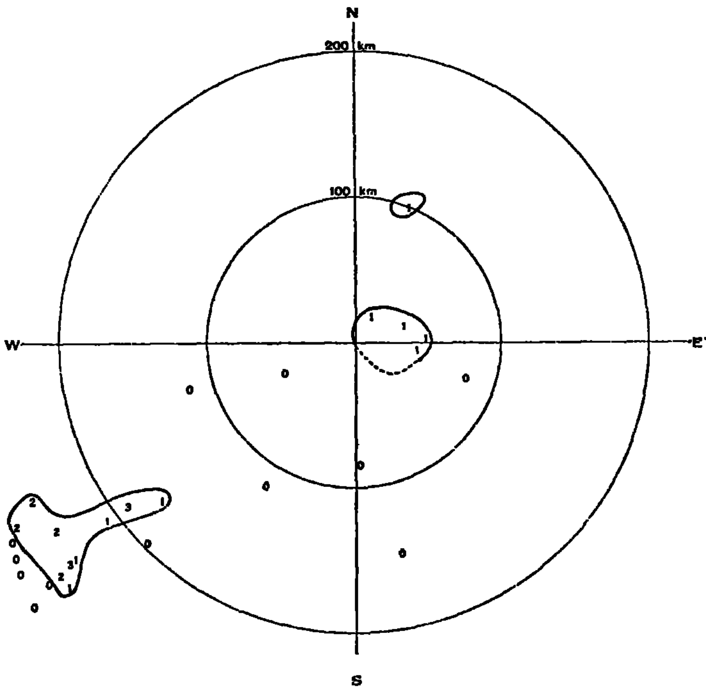

FIG. 5.—Regions of audibility of sounds of explosions of Asama, July 19, 1913. (Author's Chart XIIIa.)

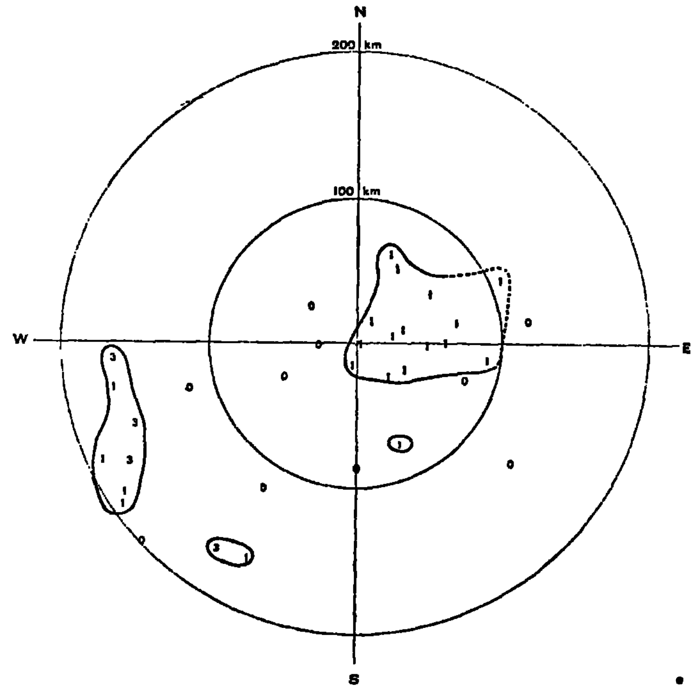

FIG. 6.—Regions of audibility of sounds of explosions of Asama, at 8h 10m on June 26, 1913. (Author's Chart VIIIa.)

and temperatures. Also when there are local convectional currents of air accompanying squalls or thunderstorms. The mathematical theory has not yet been developed for these cases, but Fujiwhara remarks that the phenomenon of rolling sound has a close relation with Helmholtzian waves.

A brief account will now be given of the way in which the shape of a region of audibility can be calculated when the meteorological conditions are known.

2. *Equations for the rays of sound.*

These were deduced by Fujiwhara in his first memoir, from the hydrodynamical equations; they simply express the fact that the ray velocity is the vector sum of the velocity of the waves relative to the air and the velocity of the wind. Let (θ, ϕ) be the spherical polar coordinates of the wave-normal at a point with rectangular coordinates x, y, z, the polar axis and axis of z being vertically upward. Then if the medium is stratified in horizontal layers and the wind has horizontal components $(u, v,)$ at the point (x, y, z) we have the equations

$$\phi = \phi_0$$

and

$$c \operatorname{cosec} \theta + u \cos \phi + v \sin \phi = c_0 \operatorname{cosec} \theta_0 + u_0 \cos \phi + v_0 \sin \phi, \tag{1}$$

which express that the velocity and direction of the line of intersection with the horizon of the tangent plane to the wave-front remain constant, u_0, v_0, c_0, θ_0 and ϕ_0 being initial values.[2]

The equations determining the rays are now:

$$dx = udt + cdt \sin\theta \cos\phi,$$
$$dy = vdt + cdt \sin\theta \sin\phi,$$
$$dz = cdt \cos\theta,$$

consequently

$$x + iy = \int_0^{z} \frac{1}{c}[c \tan\theta\, e^{i\phi} + (u + iv) \sec\theta] dz, \tag{2}$$

where θ is given by (1).

[2] *Rayleigh:* Theory of Sound. v. 2, p. 133.

3. *Conditions for reflection.*

If the wave be reflected at the level $z = \zeta$ we must have $\theta = 90°$ and so

$$c_0 \operatorname{cosec} \theta_0 - c = V \cos\alpha. \tag{3}$$

where $V\cos\alpha$ is the component along the horizontal projection of the wave-normal of the change in velocity of the wind as compared with that at the origin. In order that the wave may not be reflected at a lower level $c + V\cos\alpha$ must be greater than its value for all lower levels and so greater than its initial value c_0. We thus obtain the conditions

$$c_0 - c < V$$

and

$$\frac{d}{dz}(V\cos\alpha) > -\frac{dc}{dz}.$$

Fujiwhara replaces the last condition by the necessary condition that the vertical gradient of the horizontal wind velocity should be greater in absolute magnitude than $-\frac{dc}{dz}$.

Fujiwhara applies these conditions to an atmosphere built up of horizontal strata each a half kilometer in height, the gradients in each stratum being uniform. The constants are chosen so that the velocities at the boundaries between the strata agree with those observed at Ditcham[3] at 6:08 p. m. on May 21, 1907. The velocity of sound is calculated by using the mean temperatures for May over England.[4] In the following expressions $z, z_1, z_2, \cdots$ are measured in kilometers and each of them assumes values between 0 and $\frac{1}{2}$.

Stratum.	c.	$V\cos\alpha$.
0 —0.5	$337.0-2.2z$	$6.6\cos(259°-\phi)z$
0.5—1	$335.9-2.4z_1$	$3.3\cos(259°-\phi)+6.2\cos(33°-\phi)z_1$
1 —1.5	$334.7-2.4z_2$	$2.5\cos(322°-\phi)+6.6\cos(113°-\phi)z_2$
1.5—2	$333.5-2.6z_3$	$1.6\cos(71°-\phi)+3.6\cos(115°-\phi)z_3$
2 —2.5	$332.2-3z_4$	$3.2\cos(98°-\phi)+16.2\cos(41°-\phi)z_4$

[3] *Cave*, op. cit., p. 95.
[4] *Gold, E.* The international kite and balloon ascents. Geophys. mem. No. 5, p. 83. (Meteorol. Office, London.) 1913.

Since $c_0 - c > V$ for $z = 1.5$, there is no reflection at this level. Above $z = 2$ sound waves can be reflected because of the sudden increase of wind velocity and since the relative wind velocity has an azimuth of 56° the reflection takes place chiefly in a northeasterly direction. For the lower stratum $(z < 1)$ the azimuth is about 259° and so the reflection takes place mainly in a westerly direction. Thus the sound is audible in a region extending somewhat to the west and probably in a detached region toward the northeast.

For $\phi = 259°$ we find that $c + V\cos\alpha$ increases with z up to $z = 0.5$ and then decreases; hence among the rays with azimuth 259° one which has its vertex at a height of 0.5 kilometer has the maximum range. For this ray

$$c_0 \operatorname{cosec} \theta_0 = 337 \operatorname{cosec} \theta_0 = 335.9 + 3.3 = 339.2.$$

The range is easily found from (2) to be 17.6 kilometers. Similarly for rays with azimuth $259° \pm 30°$, $c + V\cos\alpha$ has a maximum value at $z = 0.5$, and the rays with vertices at this height are given by

$$337 \operatorname{cosec}\theta_0 = 335.9 + 3.3 \cos 30° = 338.8.$$

The maximum range, R, is 19.6 kilometers. Similarly for rays whose azimuth is 210° from the north $R = 24.8$ kilometers.

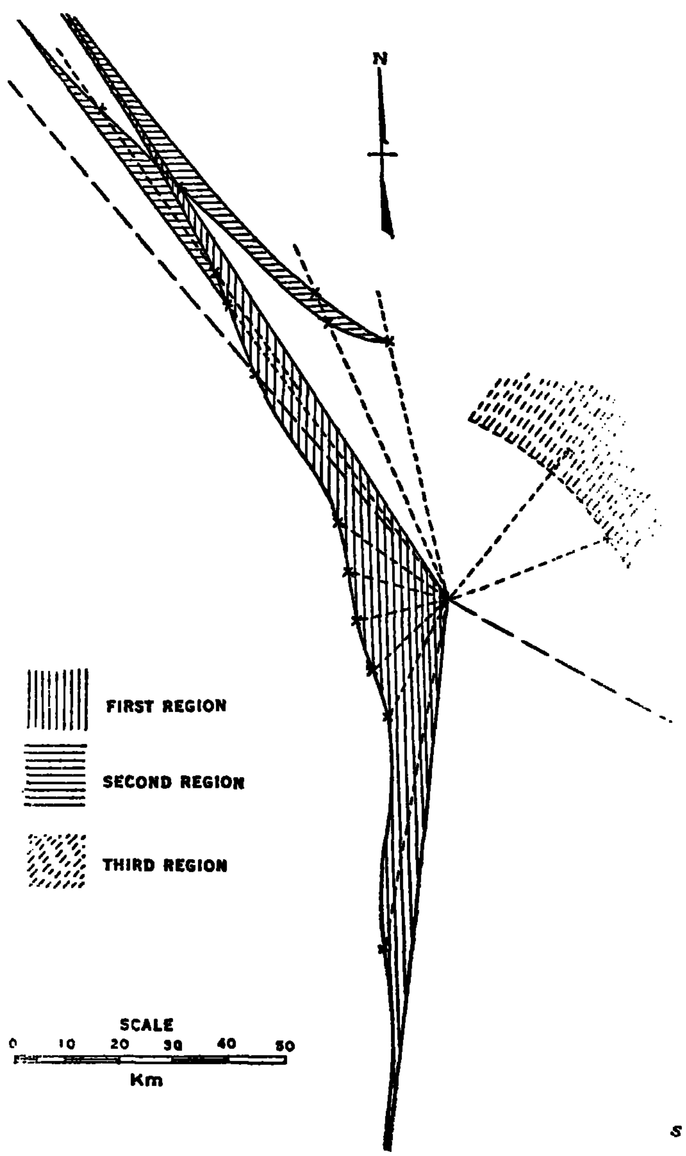

FIG. 7.—The calculated regions of audibility. (Author's fig. 2, p. 15.)

Calling the region of audibility associated with the wind in the stratum $z < 0.5$ *the first region*, we can find the extreme values of ϕ for this region by making $c + V\cos\alpha$ independent of z. The resulting equation

$$6.6\cos(259° - \phi) = 2.2$$

gives two values of ϕ, viz, $\phi_e' = 188°\ 30'$ and $\phi_e'' = 329°\ 30'$. We have $R = 0$ for $\phi = \phi_e'$ and $\phi = \phi_e''$, while $R = \infty$ for $\phi = \phi_e' + \epsilon$ and $\phi = \phi_e'' - \epsilon$, ϵ being a small positive angle.

The second region of audibility is associated with winds in the layer $0.5 < z < 1$. In order that reflection may take place $c + V\cos\alpha$ must increase with z and so ϕ must satisfy the inequality $6.2\cos(33° - \phi) > 2.4$, which gives

$$-34°\ 15' < \phi < 100°\ 15'.$$

The maximum value of $c + V\cos\alpha$ for any of these values of ϕ is attained when $z = 1$ and must be greater than c_0 or 337. This gives us the inequality

$$334.7 + 2.5\cos(322° - \phi) > 337,$$

therefore $\phi < -11°$ and so $-34°\ 15' < \phi < -11°$.

We have $R = \infty$ for the smallest value of ϕ, the corresponding value of R for the first region is found to be 66.6 kilometers.

The third region of audibility is associated with the winds above $z = 2$. In order that $c + V\cos\alpha$ may increase with z we must have $16.2\cos(41° - \phi) > 3$, therefore $321°\ 40' < \phi < 480°\ 21'$. For a ray with azimuth ϕ the minimum height ζ_m of the vertex is given by

$$4.8 + 3\zeta_m - 3.2\cos(98° - \phi) - 16.2\cos(41° - \phi)\,\zeta_m = 0.$$

Giving ϕ a value ϕ_m for which ζ_m has its minimum value $(\zeta_m)_m$, it is found by a graphical method that $\phi_m = 70°$, $(\zeta_m)_m = 2.17$ km. The corresponding range R is found to be 44.3 kilometers. A smaller range, $R = 31.5$ kilometers, is given by taking $340\sin\theta_0 = 337$. For $\sin\theta_0 = 1$ the range is 44.4 km and for $345\sin\theta_0 = 337$ it is 45.2 km. Since these two ranges are approximately equal and very nearly in the same direction we may conclude that a double report will be heard in the neighborhood of a point distant 45 kilometers from the origin and in azimuth 430°. It should be remarked that Fujiwhara assumes in his calculations that the gradient of wind velocity above 2.5 kilometers is in azimuth 430°. Below this level the wind velocity does not exceed 4 meters per second and so in an interval of about 135 seconds the cross wind can not deviate the rays more than two or three hundred meters from a line in azimuth 430°. The three regions of audibility are shown in figure 7.

The time the sound takes to describe the range R is given by the formula

$$t = 2\int_0^{\zeta} \frac{dz}{c} \sec\theta,$$

where θ is given by (1) and ζ by (3). Fujiwhara finds that $t_1 = 133.1$ sec., $t_2 = 137.8$ sec., thus the time interval is 2.3 seconds. Fujiwhara also makes a few remarks on the question of the flow of energy in a sound wave and studies the effect of the curvature of the earth on the range of a sound ray. He finds that the correction to allow for curvature is generally small, but in a rare exceptional case it may amount to as much as 42 per cent.